Type I intermittency in a dynamical system with dichotomous parameter change

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Abstract

In type I intermittency, simple models known for at least twenty years show that a characteristic u-shaped probability distribution is obtained for the laminar phase length. We have shown elsewhere that, for some cases of pathology, the laminar phase length distribution characteristic for type I intermittency may be obtained in human heart rate variability data. The heart and its regulatory systems are presumed to be both noisy and nonstationary. Although the effect of additive noise on the laminar phase distribution in type I intermittency is well known, neither the effect of multiplicative noise nor of nonstationarity (i.e. changes of the control parameter with the time) have been studied. In this paper, we first discuss the properties of two classes of models of type I intermittency: a) the control parameter of the logistic map is changed dichotomously from a value within the intermittency range to just below the bifurcation point and back; b) the control parameter is changed randomly within the same parameter range as in the model class a). We show that the properties of both models are importantly different from those obtained for type I intermittency in the presence additive noise as obtained by Hirsch twenty years ago. The two models help explain some of the features seen in the intermittency in human heart rate variability.

Intermittency in stationary dynamical systems is a broad area of research of constant interest (over 1000 papers published so far). The phenomenon appears whenever a dynamical system is close to one of several common types of bifurcation. The simplest kind of intermittency was classified by Pomeau and Manneville [1] as type I, II and III and shown to be due to the proximity of the system to the saddle-node, the Hopf and the reverse period doubling bifurcation, respectively. In these basic types of intermittency, the evolution with the time of the system may be divided into ranges of the time in which the behavior of the system is (almost) regular - the laminar phases - and ranges in which chaotic bursts

occur. Other, less common types of regular intermittency have also been identified [2] [3]. Chaos-chaos intermittency is due to crisis phenomena occurring in the system [1] and on-off intermittency is due to a symmetry breaking bifurcation [4]. Thus, identification of the type of the intermittency observed may yield important information about the system by specifying the bifurcations possible for its dynamics.

Identification of the type of intermittency in experiments and numerical analysis is made by studying its statistical properties [5] [6] [7] [8] where usually the dependence of the average laminar phase length $< l >$ on the value of the control parameter and the probability distribution of the laminar phase lengths $P(l)$ as a function of that parameter are studied. When the shape of the map is known or may be measured, reinjection probability density (RPD) analysis may be performed [9]. Note that, in many experimental situations, the map is unknown and, particularly in biology and astrophysics, the control parameter is not accessible and so only the distribution $P(l)$ may be studied.

Another factor important in experimental measurement is noise. Additive noise is then unavoidable and much attention to the effect of additive noise has been given on intermittency in dynamical systems [5] [7] [8] [10] [11] [12] [13] [14] [15]. The most important feature of the effect of noise on type I intermittency is that the well known, textbook, symmetric U-shape of the $P(l)$ curve is changed: the right peak of the distribution broadens, becomes smaller and gains a long tail extending towards larger laminar phase lengths [5] [7]. Notably, the distribution as a whole becomes narrower. More recently, the effect of additive noise was studied as the control parameter of a simplified, generic model of intermittency was changed from pre-bifurcation values to those above the saddle node bifurcation point and non-trivial results were obtained [8].

Recently, we studied [16] the effect of multiplicative noise on the type I intermittency which occurs close to the period-3 window in the logistic map. We showed that small, random changes of the control parameter within the bounds of intermittency broaden the $P(l)$ distribution instead of narrowing it as does the additive noise.

We have found [17] [16] that in some cases of human heart rate variability type I intermit-

tency may occur. We showed that the laminar phase lengths - defined as parts of the 24-hour long heart rate variability time series with a large standard deviation - have a probability distribution which is U-shaped albeit strongly asymmetric and with a long tail extending towards large laminar phase lengths. In many cases to the right of the main U-shape distribution there occurs a secondary peak in the distribution. Some of these distortions may be partly due to the shape of the underlying, unknown mapping involved but certainly may not be explained by the effect of additive noise often expected in living systems. Judging by the clinical data of the cases we have examined, it appears that the occurrence of intermittency in human heart rate variability is related to supraventricular or ventricular arrhythmia. This holds also when the number of ectopic beats is low enough so that from the medical point of view the recording may be qualified as normal. Arrhythmia may shift the state of the system (i.e. change its control parameters) for a certain time. To understand what effect such a change of parameters may have on a system in type I intermittency, we propose simple models based on the logistic map in which *dichotomous parametric change* is active. We show that the effects of the dichotomous change of the control parameter explain the main features of our results found in heart rate variability: the long tail outside the U-shaped main distribution as well as the secondary peaks which occur occasionally in heart rate variability data. The paper is organized as follows. Firstly, we briefly review the medical data used in this study. We next define the models with dichotomous parametric change. We then present the results obtained for heart rate variability and the results of our calculations. A brief discussion and conclusions section follows.

I. THE MEDICAL DATA

Heart rate variability data was extracted from 24-hour Holter device ECG recordings using the 563 Del Mar Avionics system at the National Institute of Cardiology (Warszawa, Poland). All data was checked by a qualified cardiologist: normal beats were detected, artifacts were deleted and arrhythmias were recognized. The data was sampled at 256 Hz.

4

The patient BLT had a permanent atrial tachycardia that was conducted to the ventricles with a varying extent of the atrio-ventricular block. No ventricular ectopy was observed. The patient FTCH had sinus and atrial heart rhythm with an extremely increased number of ventricular arrhythmia (around 50% of heart cycles were ectopic). Additionally, the presence of arrhythmia influnced the sinus or atrial rhythm due to retrograde conduction. The patient WJCK had sinus rhythm and ectopic beats (7% of all cycles) originating in the ventricles and in the atria.

II. MODELS OF NONSTATIONARY INTERMITTENCY

The logistic map model $x_{n+1} = ax_n(1 - x_n)$ where $x_n \in [0, 1]$, a is the control parameter and n the iteration index is well known [1]. This map has been widely used to model the properties of type I intermittency although this phenomenon will occur in any system in which the saddle-node bifurcation occurs at a control parameter range just below bifurcation point. We modified this basic model to generate nonstationary intermittency data.

For control parameter values larger than 3.5699456..... (the accumulation point [1]), periodic windows occur due to tangential bifurcations. These periodic windows are dense for parameter value lying between the accumulation point and $a = 4$. Type I intermittency occurs for control parameter values slightly below that at which the tangential bifurcation of the given periodic window occurs. The intermittency occurring close to the widest periodic window - the period 3 window - was chosen for this study. The critical value of the control parameter a at which the period 3 orbit occurs was determined numerically to be $a_c = 3.8284271271245$. The difficulty in assessing the critical value of the control parameter was that as the bifurcation point was approached the length of the laminar phase (i.e. the number of iterations corresponding to the periodic state between chaotic bursts) grows to infinity. At $a = a_c$ the length of the periodic orbit obtained exceeded 100 0000 iterations. The critical value of the control parameter was determined numerically because it was important for this study that - given the large accuracy of the control parameter used for the control parameter

- the position of the bifurcation point which occurs for the processor in the computer on which the calculations were performed be used in the analysis. The exact position of the bifurcation point may be slightly machine and compiler dependent due to roundoff errors.

The value of the control parameter, for which the maximum of the length of the laminar phase would attain a manageable value, was sought. After several trials, the control parameter $a = 3.828$ was assumed. At this value of the control parameter the maximum laminar phase was 44 iterations.

Nonstationarity may be introduced in many ways. Below, the following models of non-stationary intermittency were considered.

A. Dichotomous parameter change models

The dichotomous parameter change models were obtained using the standard formula for the logistic map. However, the value of the control parameter a was cycled with a period of $n + m$ iterations where n was the number of iterations for which a equaled 3.828 and m was the number of iterations for which a equaled either 3.8284271225 (model d1) or 3.828427128 (model d2). Both n and m were varied from 1 to 20. In some cases the range of m was extended to 100.

The motivation to use these models was the supposition that some of the results obtained for human heart rate variability may be due to a response of the regulatory system to premature ventricular beats (instances of arrhythmia). When such heart beats occur and the regulatory system of the heart is in the intermittency state (i.e. close to a tangential bifurcation) then the occurrence of a premature beat may shift the control parameter closer towards the bifurcation point (model d1) or slightly beyond it (model d2).

As for the random parametric noise models discussed in ref. [16], the results obtained here for the dichotomous parameter change were only weakly dependent on whether the control parameter assumed the value just below the bifurcation point (model d1) or just above it (model d2). For this reason only the calculations for model d1 will be discussed

here. Note, however, that all the calculations discussed below were also performed for model d2.

B. Dichotomous parametric noise model

In this model again the standard formula for the logistic map was used but the value of the control parameter at every iteration was changed randomly between two values: 3.828 and 3.8284271225. The model had a secondary control parameter: the probability with which these two values were chosen.

III. CALCULATION OF THE DISTRIBUTION OF THE LAMINAR PHASES

Given a time series either from measurement or obtained through calculations in a model, the laminar phases must be found and their distribution calculated. In the generic, simplified model of type I intermittency [1] the subsequent iterations during the laminar phase of intermittency have a very low standard deviation. This is because the generic model describes the behavior of the system in the narrow channel close to the bifurcation point. In an intermittency state occurring close to period-k window there will be k such channels so that, except for k = 1, the resultant time series within the laminar phases will have a large standard deviation.

We calculated the average deviation of each time series studied here within a sliding window of j data points, j = 2-10 (windows up to 100 data points were tested) [17] [16]. A histogram of the resultant time series of the window average deviation was calculated. Maxima in this histogram defined the phases i.e. those of the laminar and of chaotic bursting behavior. The minimum between such maxima was used as the discriminating criterion and - when the average deviation was larger than this the coding level - the number of iterations between the crossings of the coding level defined the length l of the laminar region. Finally, the probability density distribution $P(l)$ was calculated as a normalized histogram of l.

To use the above described algorithm one needs to define the sliding window length. Tests carried out with stationary intermittency time series calculated using the logistic map show that the window length should be approximately equal to the period of the orbit in the adjacent periodic window. If the length of the sliding window is too small compared with the period of the orbit, a Poisson-like distribution is obtained for $P(l)$ falsely indicating no intermittency. If, on the other hand, the window is too long then some averaging results and a false tail extending towards long laminar phase lengths appears. Below, all data was analyzed using windows either 3 or 5 data points in length.

IV. RESULTS

A. Intermittency in human heart rate variability

The most common type of laminar length distribution found in our studies is shown in fig. 1 (compare fig.1 in ref. [16]) for window size 3 RR intervals and in fig.2 for the patient WJCK. It was found that in most cases studied here 3 or 5 data point length were the optimal lengths of the sliding window. Larger values averaged the time series in such a way that the details of the time series were lost. It can be seen that an almost perfect shape of the probability distribution for intermittency type I was obtained in both cases except that there was a tail to the right of the characteristic U- shaped distribution. In other examples of such a shape of the distribution the tail in the distribution extended still further towards longer laminar lengths.

A double peaked distribution of laminar lengths was obtained for window lengths 3 and 5 intervals (the latter is depicted in fig.3) for the patient FTCH. Other examples of such split peak in the $P(l)$ distribution were found for different patients suffering from different forms of arrhythmia.

The results shown here similarly as those published earlier in [17] and in [16] indicate that the distribution of laminar lengths obtained from the examples of human heart rate

8

variability are very typical for such distributions obtained for the standard models of type I intermittency [1] [5] except for the following features: there is a long tail extending towards long laminar phase lengths present in all cases and in some of them also a second peak or multiple peaks are obtained. Using numerical models, two possible reasons for these additional features are examined below.

B. Intermittency in the presence of dichotomous noise

Fig.4 depicts the distribution of laminar phases for the logistic map with random dichotomous parametric noise. It can be seen that when the two values of the control parameter were chosen with an equal probability (crosshatched bars) the effect was very similar to that of the random parametric noise of ref. [16]: the distribution of laminar phases becomes broader than for the stationary case (black bars) and smaller. However, when the probability of choosing the prebifurcation value 3.8284271225 was 25 % then the $P(l)$ distribution (open bars) the shape of the stationary distribution s retained but a tail extending towards long laminar phase lengths appears.

C. Intermittency in the presence of dichotomous parameter change

For $n = 1$ and m changed from 1 upwards, it was found that for $m = 2 + 3k$, $k = 0, 1, 2, 3, ...$ the period-3 state was obtained although - without nonstationarity - for $a = 3.8284271225$ intermittency was found for at least 10^5 iterations. For other values of m the only effect of dichotomous parameter change was to increase the width of the distribution of the laminar phase lengths. Examples of such behavior are shown in fig.5 for $m = 1, 6, 10$. Similarly, for $n = 2$ with $m = 2$ or $m = 3$, itermittency with a broadened distribution of laminar phase lengths was obtained. However, for $n = 2$ and all other values of m the period-3 state was obtained without intermittency. Also for $n = 3$ and 4, although

9

no periodic states were found, the only effect of the dichotomous parameter change was to broaden the distribution for all values of m.

A new phenomenon occurred for $n = 5$. For $m < 10$, again just the broadening of the distribution was found - similar to that in fig.5. Fig.6 depicts the distributions for the stationary case (black bars), $m = 10$ (cross hatched bars) and for $m = 25$ (white bars). It can be seen that a large splitting of the right peak in the distribution occurs. A further increase of m resulted in an increase of the width of the distribution but the right hand peaks decreased in size and drew closer together. At $m = 75$ the maximum phase length was slightly less than 225 iterations but right peak was very broad and its splitting was only weakly visible.

For $n = 10$ and $m = 2$ a strong splitting of the right peak was observed (Fig.7). It can be seen that the right peak of this distribution for the nonstationary case is split into two maxima in such a way that the left of these falls somewhat to the left of the maximum laminar phase length for the stationary case. Surprisingly for $m = 3$ no splitting occurred but the distribution had the same shape as for the stationary case. However, the maximum laminar phase length was slightly less than for $m = 2$ (not shown). For $m = 5$, a splitting similar to that obtained for $m = 2$ occurred with the distribution broadening up to laminar phase length 60. A further increase of m resulted in a decrease of the splitting with a simultaneous broadening of the distribution and of its right hand peak so that at $m = 30$ the splitting was only barely visible.

V. DISCUSSION AND CONCLUSIONS

Recently, we have demonstrated that 24-hour human heart rate variability may exhibit the main statistical features of type I intermittency [17] [16]. We present here two kinds of data. The first are three characteristic examples of type I intermittency found by us in heart rate variability measurements. The second are the results of calculations in simple models of type I intermittency in a nonstationary system. The results obtained form these models

seem to be interesting in themselves: new properties of a system in type I intermittency with parametric noise and under the action of a dichotomous parameter change were obtained. However, an important motivation in defining these models was the attempt to understand the reasons for certain deviations of the properties of the intermittency found in heart rate variability from the textbook descriptions [1]. Two reasons for these deviations were considered: the effect of dichotomous parametric noise and the effect of abrupt, dichotomous control parameter change such as may be due to episodes of arrhythmia.

The models considered here are based on the well known logistic equation which has been widely used as the primary model of type I intermittency. These models are of two kinds: in the first, a random, multiplicative, dichotomous noise is applied in such a way that the system approaches randomly very close to the bifurcation point. Such a model would correspond to arrhythmia occurring erratically in the time. In the second category of models, the control parameter is dichotomously cycled with the period n+m: n iterations at a predefined control parameter value inside the intermittency range and the next m iterations at a control parameter value either very close the bifurcation point but in intermittency or just inside the periodic window. It was found that only small quantitative differences were obtained depending on whether the system goes through the saddle-node bifurcation or not. Such a class of models would correspond to an arrhythmia occurring in a regular way, every certain number of normal heartbeats.

Comparison of the properties of the models with the results obtained for human 24-hour heart rate variability show that a) the long tails in the laminar length probability density distributions are most probably due to multiplicative (parametric) noise and not to additive noise, b) the double or triple peaks in such distributions found in some cases for heart rate variability are due to the interplay between the number of natural heart beats and the number of ectopic beats (arrhythmia). Note that, for the extremely simple models presented here, even very few iterations of the system with the control parameter in the vicinity of the bifurcation point interspersed between several iterations performed far away from that point introduce a strong splitting of the peaks of the laminar phase lengths distribution. Although

no discrete map models of the complete heart rate variability control system exist, nonlinear dynamics shows that many features found in simple models such as the logistic map are universal and are often found in more elaborate models. Note, that the circle map [1] is most probably a more adequate class of maps to describe the properties heart rhythm [18]. However, we need to remember that type I intermittency occurs also in circle maps and that the basic properties of type I intermittency are map independent. In choosing the logistic map as the core of the nonstationary models discussed here we have avoided the cumbersomeness of two-parameter models.

The results presented here seem to indicate that the heart rate variability regulating system - at least for the types of pathology studied in the examples given - may be permanently close to a saddle-node bifurcation.

VI. ACKNOWLEDGMENTS

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VII. FIGURE CAPTIONS

Fig.1 The laminar phase length probability density found for the 24-hour heart rate variability recording for the patient BLT.

Fig.2 The laminar phase length probability density found for the 24-hour heart rate variability recording for the patient WJCK.

Fig.3 The laminar phase length probability density found for the 24-hour heart rate variability recording for the patient FTCH. Note the characteristic splitting of the righthand peak of the distribution.

Fig.4 Distribution of laminar phases for the logistic map with random dichotomous parametric noise: black bars 85 % probability of a=3.828, crosshatched bars 50 % probability of a=3.828. White bars - stationary case without noise.

Fig.5 Distribution of laminar phase lengths obtained for the the dichotomous parametric change model with $n = 1$ and $m = 1, 6, 10$ as marked and for the sationary case S. The left maxima of all distributions coincide.

Fig.6 Distribution of laminar phase lengths with a split righthand peak obtained for the the dichotomous parametric change model with $n = 5$ and $m = 10$ (cross hatched bars) and for $m = 25$ (white bars). The black bars depict the result for the statioanry case. Peaks at right maxima are labelled with the value of m and the left maxima of all distributions coincide.

Fig.7 Distribution of laminar phase lengths with a split righthand peak obtained for the the dichotomous parametric change model with $n = 10$ and $m = 2, 5$ and 25 (cross hatched,

14

white and horizontally hatched bars, respectively). Stationary case - black bars.